Cuadernos de Neuropsicología / Panamerican Journal of Neuropsychology
2026, Vol. 20 N.º 1 126 - 145

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

ENHANCING PERCEPTUAL REASONING THROUGH ACTIVE CALCULUS INSTRUCTION: A NEUROPSYCHOLOGICAL PILOT STUDY IN ENGINEERING EDUCATION

MEJORANDO EL RAZONAMIENTO PERCEPTIVO MEDIANTE LA ENSEÑANZA ACTIVA DEL CÁLCULO: UN ESTUDIO PILOTO NEUROPSICOLÓGICO EN EDUCACIÓN EN INGENIERÍA

APRIMORANDO O RACIOCÍNIO PERCEPTIVO POR MEIO DO ENSINO ATIVO DE CÁLCULO: UM ESTUDO PILOTO NEUROPSICOLÓGICO NA EDUCAÇÃO EM ENGENHARIA

RECIBIDO: *23 septiembre 2025* ***ACEPTADO:*** *26 diciembre 2025*

Raúl D. Jiménez [1] https://orcid.org/0000-0001-6429-9820 **Susana Arancibia-Carvajal** [2] https://orcid.org/0000-0003-2975-3260
Walter Terrazas-Nuñez [2] https://orcid.org/0000-0002-5307-0403 **Ema Huerta** [3] https://orcid.org/0000-0001-6676-1820
Nathalia Herrán [2] https://orcid.org/0009-0006-5267-2935 **Loreto Fuentes** [2] https://orcid.org/0009-0008-6878-483X
Marcos Domic-Siede [2] https://orcid.org/0000-0002-5191-1133

1. Departamento de Matemáticas, Universidad Católica del Norte, Antofagasta, Chile.
2. Escuela de Psicología, Facultad de Humanidades, Universidad Católica del Norte. Antofagasta, Chile.
3. Unidad de Innovación Docente de las Ingenierías (UIDIN), Universidad Católica del Norte. Antofagasta, Chile.

Keywords: *Calculus; Education; Cognitive Functions; Visuospatial abilities.*

Palabras clave: *Cálculo; Educación; Funciones cognitivas; Habilidades visoespaciales.*

Palavras-chave: *Cálculo; Educação; Funções cognitivas; Habilidades visuoespaciais.*

ABSTRACT

Calculus, a cornerstone of mathematics that explores continuous change through differentiation and integration, is pivotal for higher education in engineering and STEM careers. Yet, its instruction presents challenges, with many students grappling with its complexity, accentuating the need for innovative teaching strategies. Grounded in the nexus between calculus and higher cognitive functions, this research evaluates the outcomes of a student-centered approach in enhancing understanding and applying calculus and seeks to determine its impact on broader cognitive functions. Employing a pretest-retest design with control and intervention groups, we merge cognitive evaluations, including subtests from the Wechsler Adult Intelligence Scale, with calculus performance assessments. Our results revealed the intervention group exhibited enhanced calculus performance and improvement in the perceptual reasoning index, especially in block design subtest, reflecting enhanced visuospatial skills. This study offers valuable perspectives for improving calculus instruction and potentially improving students' cognitive capabilities by implementing a student-centered approach.

Corresponding Author: Marcos Domic-Siede, Av. Angamos 0610, Escuela de Psicología, Universidad Católica del Norte, Antofagasta, Chile; email: mdomic@ucn.cl

This study was financially supported by the DGPRE of the Universidad Católica del Norte.

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

R E S U M E N

El Cálculo, piedra angular de las matemáticas que estudia el cambio continuo mediante la diferenciación y la integración, es fundamental en la educación superior en ingeniería y carreras STEM. Sin embargo, su enseñanza presenta desafíos, ya que muchos estudiantes lidian con su complejidad, lo que acentúa la necesidad de estrategias innovadoras de instrucción. Basada en la relación entre el cálculo y las funciones cognitivas superiores, esta investigación evalúa los resultados de un enfoque centrado en el estudiante para mejorar la comprensión y aplicación del cálculo, y busca determinar su impacto en funciones cognitivas más amplias. Utilizando un diseño de pretest-retest con grupos control e intervención, combinamos evaluaciones cognitivas, incluyendo subpruebas de la Escala de Inteligencia de Wechsler para Adultos, con pruebas de desempeño en cálculo. Nuestros resultados revelaron que el grupo de intervención mostró un mejor desempeño en cálculo y mejoras en el índice de razonamiento perceptivo, especialmente en la subprueba de diseño con cubos, lo que refleja un fortalecimiento de las habilidades visoespaciales. Este estudio ofrece perspectivas valiosas para optimizar la enseñanza del cálculo y, potencialmente, para favorecer el desarrollo de las capacidades cognitivas de los estudiantes mediante la implementación de un enfoque centrado en el estudiante.

R E S U M O

O Cálculo, pedra angular da matemática que estuda a mudança contínua por meio da diferenciação e da integração, é fundamental no ensino superior em engenharia e carreiras STEM. No entanto, seu ensino apresenta desafios, pois muitos estudantes enfrentam dificuldades com sua complexidade, o que acentua a necessidade de estratégias inovadoras de instrução. Com base na relação entre o cálculo e as funções cognitivas superiores, esta pesquisa avalia os resultados de uma abordagem centrada no estudante para aprimorar a compreensão e a aplicação do cálculo, e busca determinar seu impacto em funções cognitivas mais amplas. Utilizando um delineamento de pré-teste e pós-teste com grupos controle e intervenção, combinamos avaliações cognitivas, incluindo subtestes da Escala de Inteligência Wechsler para Adultos, com provas de desempenho em cálculo. Nossos resultados revelaram que o grupo de intervenção apresentou melhor desempenho em cálculo e melhorias no índice de raciocínio perceptivo, especialmente no subteste de construção com blocos, refletindo um fortalecimento das habilidades visuoespaciais. Este estudo oferece perspectivas valiosas para otimizar o ensino do cálculo e, potencialmente, favorecer o desenvolvimento das capacidades cognitivas dos estudantes por meio da implementação de uma abordagem centrada no aluno.

Calculus is a branch of mathematics that studies continuous change, primarily through the concepts of differentiation and integration. Differentiation addresses the rate of change (like finding the slope of a curve at a point), while integration deals with finding the quantity of things when we know the rate of change (like calculating the area under a curve) (DeBaggis & Miller, 1966; McQuarrie, 2003). Given its foundational principles, calculus is crucial for higher education training in engineering (Kreyszig, 2011) and remains a linchpin for various Science, Technology, Engineering, and Mathematics (STEM) careers, serving as the mathematical backbone for understanding and modeling a wide array of real-world systems and phenomena (Hughes-Hallett et al., 2013). Nevertheless, the pedagogical landscape for calculus has not always kept pace with its importance, revealing gaps in how it is taught and comprehended.

Over the past 30 years, significant global changes have reshaped the roles and approaches of higher education institutions, necessitating adaptations in methodologies and study programs (García, 2013). Especially in STEM careers, such as Engineering, the teaching of calculus has shown a concerning trend towards algebraization and arithmetization, moving away from its practical application (Artigue, 1998; Camarena, 2010). This decontextualized approach to calculus presents challenges for students, hindering their comprehensive training in engineering and potentially leading to discouragement and dropping out (Ellis et al., 2021; Bressoud et al., 2015; Karunakaran & Higgins, 2021). In general, mathematical teaching has been axiomatized and routine, often disconnected from everyday life and real situations (Cordero, 2007; Moreno, 2005). This scenario emphasizes the importance of contextualized learning, where calculus and other mathematical areas are taught through real-world problems, promoting the integration and application of knowledge in the professional realm (De Corte & Verschaffel, 2008; Fraihat et al., 2022).

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

In the local context, data from the Programme for International Student Assessment (PISA) show that Chilean secondary students perform below the OECD average in mathematics (412 vs. 472 points; Agencia de Calidad de la Educación, 2022). At the tertiary level, results from the Higher Education Access Test (PAES) reveal substantial performance gaps linked to the type of high school attended, with differences averaging 143 points in reading proficiency and 192 points in mathematics (Departamento de Evaluación, Medición y Registro Educacional, 2024). These disparities are reflected in university-level mathematics courses, where failure rates in algebra and calculus often approach 60%, particularly during the first years of STEM programs, negatively impacting academic progression and retention (Hagman et al., 2017).

A persistent challenge in calculus education is the predominance of highly formal and decontextualized instructional approaches, often emphasizing symbolic manipulation over conceptual understanding and practical application (Artigue, 1998; García, 2013). This emphasis may limit students' ability to connect mathematical concepts with real-world problem solving and can contribute to disengagement and high failure rates in early STEM courses.

A growing body of research indicates that active and student-centered learning approaches improve conceptual understanding, engagement, and long-term retention compared to traditional lecture-based instruction (Bonwell & Eison, 1991; Dresel et al., 2024; Duran et al., 2022; Freeman et al., 2014; Keiler, 2018; Lugosi & Uribe, 2020; McCarthy & Anderson, 2000; Prince, 2004; Sutherland & Bonwell, 1996). In mathematics education, strategies such as contextualized problem-solving, problem-based learning, flipped classroom models, and technology-enhanced instruction have been shown to promote deeper conceptual processing and stronger connections between abstract concepts and real-world applications (Camacho-Machín & Guerrero-Ortiz, 2015; Cevikbas & Kaiser, 2020; Dockendorff & Zaccarelli, 2024; Johnson, 2002; Lam, 2007; Mazzeo et al., 2003; Sokolowski et al., 2011; Shé et al., 2023; Spooner, 2023).

Other widely used approaches—including Team-Based Learning, scaffolded instruction, and Just-in-Time Teaching (JiTT)—have demonstrated positive effects on student participation, feedback cycles, and collaborative problem solving (Burgess et al., 2020; Hmelo-Silver, 2004; Holton & Clarke, 2006; Novak et al., 1999; Patterson et al., 2003; Savery, 2006; Naughton et al., 2020). Within this framework, the present study incorporates contextualized exercises, JiTT, and formative assessment strategies to support student preparation and adaptive instruction (Black & Wiliam, 2009; Bozzi et al., 2021; Cheung & Slavin, 2013; Kozma, 2003; Pintor et al., 2014). These instructional approaches may be particularly relevant in calculus because the subject places substantial demands on executive functions such as working memory, cognitive flexibility, and visuospatial reasoning.

Beyond instructional strategies, mathematical performance is also strongly shaped by individual cognitive functions. Research has identified key processes—such as visuospatial abilities, working memory, sustained attention, inhibition, and processing speed—as central to mathematical competence (Dehaene et al., 1999; Geary & Hoard, 2005; Keogh, 1994; McLean & Hitch, 1999; Rourke, 1993; Share et al., 1988). In STEM contexts, working memory in particular has emerged as a robust predictor of success in mathematics-intensive courses (Berkowitz et al., 2022), highlighting the need to consider both instructional design and cognitive factors when addressing performance gaps.

Central to understanding these cognitive functions is the concept of executive functions. Adele Diamond, a leading researcher in neuropsychology, delineates executive functions as a set of essential cognitive skills paramount for conscious thought, emotional regulation, and decision-making (Diamond, 2013). As Diamond (2013) articulated, executive functions can be categorized into three core components: *i)* Working memory: This entails the temporary retention and manipulation of information (Santa-Cruz & Rosas, 2017). Calculus often requires holding several pieces of information in mind simultaneously, like when solving a multi-step integral or applying the chain rule in differentiation. An effective working memory is crucial for managing these pieces and connecting them cohesively. *ii)* Inhibition: Pertains to the capability to suppress impulses and resist distractions to focus on the task at hand. While solving calculus problems, students often encounter multiple potential solution paths or need to choose between various formulas and techniques. The ability to inhibit irrelevant or less effective strategies and focus on the most appropriate one is crucial. The third component is *iii)* Cognitive flexibility: This is the skill to switch from one task to another or adjust to new demands or rules (Davidson et al., 2006). The study of calculus is replete with abstract concepts and various techniques. Being able to switch between different

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

modes of thinking, like moving from a graphical understanding of a problem to an algebraic one, demonstrates cognitive flexibility.

These three central components contribute to higher executive functions (Evans et al., 1993; Domic-Siede et al., 2024): *iv)* Fluid intelligence (reasoning and problem-solving): A crucial component, this refers to the capacity for logical thought, problem-solving, and identification of patterns, especially in novel situations (Morris & Ward, 2005). Fluid intelligence is central to grasping new calculus concepts, recognizing patterns, and applying logical thought to problem-solving, especially when encountering unfamiliar problems; and *v)* Planning: This is the forward-thinking ability to organize actions, anticipate outcomes, and strategize for future tasks (Collins & Koechlin, 2012; Domic-Siede et al., 2021, 2022, 2023; Hayes-Roth & Hayes-Roth, 1979; Lezak, 1995). Solving calculus problems, especially more complex ones, often necessitates a multi-step approach. The ability to plan, strategize, and organize one's thoughts and actions is critical for success.

The Wechsler Adult Intelligence Scale (WAIS) is a comprehensive tool that allows the measurement of fluid intelligence, a capacity crucial for reasoning, problem-solving, and abstract thinking. Furthermore, the WAIS subtests associated with working memory, perceptual reasoning, and processing speed align closely with executive functions (Aarnoudse-Moens et al., 2013). The working memory capacity enables individuals to hold and juggle information short-term, essential for tasks like problem-solving and logical reasoning. Calculus often requires holding multiple pieces of information simultaneously, such as during multi-step problem-solving processes. The digit span and arithmetic subtests of the WAIS assess this ability, which is essential for following complex mathematical procedures and calculations. For instance, studies, such as those by Cragg et al. (2017) and Caviola et al. (2012), indicate that students with better working memory tend to perform better in mathematics, as they can handle the cognitive load required for multi-step problem-solving and calculations in calculus (Van Bueren et al., 2022; Visu-Petra et al., 2014).

Perceptual reasoning relates to the ability to interpret and organize visual-spatial information, which is intrinsically linked with cognitive flexibility and working memory. This skill plays a role when visualizing calculus problems, especially when interpreting or sketching graphs of functions, understanding the behavior of curves, or grasping geometric interpretations of integration and differentiation. This ability is crucial in calculus for understanding geometric interpretations of functions, limits, and derivatives. Tasks such as block design and matrix reasoning in the WAIS align closely with these cognitive demands. Research has shown that spatial reasoning is a significant predictor of mathematical achievement. Studies indicate that fluid reasoning, which involves the ability to solve novel problems and is closely related to spatial reasoning, consistently correlates with mathematical performance across various age groups (Schneider & McGrew, 2012; Green et al., 2017). Furthermore, spatial reasoning skills are essential for tasks requiring the visualization of mathematical concepts and problem-solving in STEM fields (Battista, 2007; Mix et al., 2016). In particular, Battista (2017) found that enhancing students' spatial reasoning abilities can lead to improved performance in mathematics, including calculus. This supports the notion that interventions aimed at developing these cognitive skills can have a positive impact on students' understanding and application of complex mathematical concepts.

Lastly, processing speed, while not an executive function per se, intertwines with the efficiency of executive functions. It reflects how swiftly one can exercise inhibition and utilize working memory in fast-paced, demanding tasks (Rosas et al., 2014; Wechsler, 2008). While deep understanding is paramount in calculus, the ability to quickly process information can be beneficial, especially in timed testing environments or when working through longer computations. The ability to quickly and accurately process information is beneficial in calculus, partcularly under time constraints during tests and problem-solving activities. The symbol search and coding subtests of the WAIS measure this aspect of cognitive function. Research indicates that processing speed is a significant predictor of mathematical performance, including calculus. Faster processing speed allows individuals to efficiently handle and integrate complex information, which is essential for solving multi-step calculus problems. Studies, such as those by Cheng et al. (2021) and Peterson et al. (2023), demonstrate that cognitive processing speed predicts performance in mathematical tasks due to the need for quick and accurate manipulation of information.

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

Such skills evaluated by the WAIS subtests offer valuable insight into an individual's executive function workings and their capability to confront complex cognitive challenges, notably those encountered in mathematical and broader STEM domains. In essence, the rigorous nature of calculus demands a blend of cognitive functions for effective understanding and problem-solving. The subject's inherent complexity requires students to synthesize abstract concepts, apply logical reasoning, and often visualize mathematical scenarios, making the executive functions and cognitive abilities measured by the WAIS relevant to mastering calculus. For detailed examples of how cognitive functions are implemented in major topics of calculus (elements of real numbers, conics, functions, limits and continuity, derivatives, and their applications), see **Supplementary Table S1**.

Given the inherent demands of calculus and the cognitive skills it requires, a critical question emerges: How effective is a student-centered approach in fostering the abilities essential for mastering calculus? Our research addresses this question by bridging pedagogical strategies with the cognitive abilities they are intended to develop. Specifically, we evaluated a student-centered teaching method—integrating contextualized exercises, formative assessment, and JiTT—to determine its impact on both calculus performance and cognitive processes associated with executive functioning, particularly working memory and visuospatial reasoning.

This dual focus seeks to provide educators and students with evidence-based insights into effective instructional methodologies, while also clarifying their broader cognitive benefits. Through this lens, our study aims to contribute to a transformation of calculus instruction—promoting deeper conceptual understanding, enhancing problem-solving competencies, and ultimately strengthening the cognitive capacities that underpin success in STEM fields.

MATERIALS AND METHODS

Participants

This study has a quasi-experimental design with an experimental (intervention) and control group. Data were collected from 85 college students (24 females) between 18 and 20 years old (mean age = 18.42; SD = 0.66) belonging to the Universidad Católica del Norte engineering careers. From the total sample, 46 participants were from the control group (11 females) and 39 were from the intervention group (13 females). Due to logistical constraints associated with the administration time of the WAIS-IV subtests, cognitive assessments were conducted on a voluntary subsample of participants from each group. Specifically, from both the intervention group and the control group, we took subsamples consisting of 29 and 22 students respectively, to whom additional cognitive tests were administered (**Table 1**).

Table 1.
Sociodemographic Data

	All group		Intervention Group		Control Group		Intervention subsample		Control subsample	
	n	Mean age	n	Mean age	n	Mean age	n	Mean age	n	Mean age
Female	24	18.54	13	18.77	11	18.27	11	18.46	7	18.43
Male	61	18.38	26	18.50	35	18.29	18	18.44	15	18.47
Total	85	18.42	39	18.59	46	18.28	29	18.44	22	18.55

An a priori power analysis conducted using G*Power (Faul et al., 2007) indicated that a minimum total sample of 34 participants would be required to detect medium-sized effects ($f = 0.25$) in a repeated-measures ANOVA design with two groups and two measurements, assuming $\alpha = .05$ and power = .80. The cognitive subsample included in the present study ($N = 51$) exceeded this threshold. Additionally, a sensitivity power analysis indicated that, with the available sample size and $\alpha = .05$, the study had sufficient power (.95) to detect medium-sized effects ($f \approx 0.26$). However, smaller effects may not have been detected.

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

The Ethics Committee of the Universidad Católica del Norte approved procedures, and all participants signed an informed consent form before the beginning of the study (approval ID number: 038/2022).

Instruments

In the study, learning outcomes focused on calculus skills were assessed through three tests and a workshop conducted throughout the course.

To assess cognitive performance in higher-order functions, we used ten subtests from the Wechsler Intelligence Scale for Adults-Fourth Chilean Edition (WAIS-IV). These subtests are part of the working memory and perceptual reasoning indices. They were administered to a subsample of the groups both at the beginning and end of the course. The WAIS-IV is a widely recognized cognitive assessment tool and has demonstrated strong psychometric properties (Rosas et al., 2014).

Calculus tests

The following learning outcomes were assessed: *i)* Apply the properties of real numbers; *ii)* Solve problems involving conic curves; *iii)* Calculate the limit of indeterminate forms of real functions in one variable; *iv)* Determine injectivity and/or continuity of real functions in one variable; *v)* Calculate the derivative of real functions in one variable; *vi)* Interpret the derivative in physical and geometric problems; *vii)* Sketch the graph of real functions in one variable; and *viii)* Solve optimization problems of real functions in one variable. See **Supplementary Material 14.1**.

Students were evaluated based on a weighted average grade from three tests and one workshop, each having a grading scale ranging from 1 to 7. Each test and the workshop had a distinct weightage in the final grade: the first test comprised 25%, the second 30%, the third 35%, and the workshop 10%. A minimum weighted average grade of 4 is required to pass, and students are also required to achieve a level of proficiency of at least 60% to meet the approval requirement for the course.

Perceptual Reasoning Index

Block Design: This subtest consists of the presentation of three-dimensional models using red and white cubes. The participant uses a number of cubes to recreate models that are presented to them by the evaluator within a time limit. This test evaluates visuoperceptual coordination, perceptual processing, mainly the integration of visuospatial information and the generation of strategies to manipulate visuoperceptual information. Constructing designs using colored blocks requires switching strategies and perspectives to match the pattern shown and involves planning, as participants must determine the sequence of moves to replicate the design within a time limit (Rosas et al., 2014; Carroll, 1993; Kaufman & Lichtenberger, 1999, 2006; Groth-Marnat, 2009; Lichtenberger & Kaufman, 2009; Sattler & Ryan, 2009).

Matrix Reasoning: In this subtest, the participant is asked to complete a matrix or serial reasoning problem by selecting the missing section among five answer options. Items are not timed. This test measures fluid intelligence, visuospatial ability, simultaneous processing, and perceptual organization. Participants complete visual patterns and sequences, requiring the ability to shift between different types of reasoning and adapt to new rules (Rosas et al., 2014; Kaufman & Lichtenberger, 1999, 2006; Groth-Marnat, 2009; Lichtenberger & Kaufman, 2009; Sattler & Ryan, 2009).

Visual Puzzles: Participants are asked to select three answer choices (out of six) that could combine to reproduce a geometric image. Items are timed. It is a non-motor task that measures perceptual reasoning, visuospatial ability, visual analysis and synthesis abilities, and simultaneous processing (Rosas et al., 2014; Kaufman & Lichtenberger, 1999, 2006; Groth-Marnat, 2009; Lichtenberger & Kaufman, 2009; Sattler & Ryan, 2009).

Figure Weights: Participants are shown a visual display on a page that typically consists of a scale (or balance) with one side weighted by one or more shapes and the other side left blank. The goal of the participant is to select from several given options the shape (or combination of shapes) that will balance the scale. The shapes often have assigned numerical values based on their sizes or the symbols contained within them. By determining these values, the participant can deduce the correct weight needed to balance the scale. The execution time for this task is registered. This subtest measures fluid

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

reasoning, more specifically, quantitative and analogical reasoning, the ability to perceive the underlying relationships among visual objects and to use reasoning to deduce the missing value that would maintain balance or equilibrium, requiring logical reasoning and problem-solving skills (Rosas et al., 2014; Wechsler, 2008).

Picture Completion: This subtest involves presenting an image of an object or scene missing a part. The participant must identify the missing part within 20 seconds. This subtest evaluates visual perception, perceptual organization, and attention to visual details (Rosas et al., 2014; Kaufman & Lichtenberger, 1999, 2006; Groth-Marnat, 2009; Lichtenberger & Kaufman, 2009; Sattler & Ryan, 2009).

Working Memory Index

Digit Span: This subtest includes three tasks: Digit Forward, Digit Backward, and Digit Sequencing. In Digit Forward, the participant repeats the numbers said by the examiner. In Digit Backward, the participant is required to repeat the numbers in the reverse order that was presented. In Digit Sequencing the individual is required to sequence or order the numbers from smallest to largest. Although there is no time limit for the individual to answer, the examiner reads each number at the rate of one number per second. The working memory demands for the Digit Backward and Sequencing tasks are greater than those for the Digit Forward task, but all three tasks are designed to measure working memory. Digit span also assesses auditory processing, short-term auditory memory, and attention (Rosas et al., 2014; Reynolds, 1997; Groth-Marnat, 2009; Sattler & Ryan, 2009).

Letter-Number Sequencing: The participant is verbally given a series of numbers and letters in a mixed sequence. Their task is to first repeat the numbers in ascending order and then the letters in alphabetical order. For example, if the examiner says "B, 3, A, 2", the correct response would be "2, 3, A, B". The difficulty in this subtest increases as the sequences become longer and involve more numbers and letters to rearrange. This subtest measures working memory, the ability to temporarily hold and manipulate information (Rosas et al., 2014; Kaufman & Lichtenberger, 1999, 2006; Groth-Marnat, 2009; Lichtenberger & Kaufman, 2009; Sattler & Ryan, 2009).

Arithmetic: Items in this subtest require the participant to mentally solve word arithmetic problems within a time limit. The Arithmetic subtest measures working memory, attention, sequential processing, and numerical reasoning. Solving complex problems requires planning steps and strategies to arrive at the correct solution within the time limit (Rosas et al., 2014; Kaufman & Lichtenberger, 1999, 2006; Groth-Marnat, 2009; Lichtenberger & Kaufman, 2009; Sattler & Ryan, 2009).

Processing Speed Index

Symbol Search: This subtest necessitates the participant's ability to swiftly identify two target symbols within a sequence of various symbols. Employing a pencil, individuals mark the corresponding symbol or utilize a designated "no" box to denote their responses. A span of 120 seconds is allocated to traverse as many sequences (items) as feasible. This procedure taps into visuomotor processing speed, short-term visual memory, visual discrimination, and sustained attention. The ability to focus on relevant targets while ignoring distractions is crucial, reflecting inhibitory control (Rosas et al., 2014; Kaufman & Lichtenberger, 1999, 2006; Groth-Marnat, 2009; Lichtenberger & Kaufman, 2009; Sattler & Ryan, 2009).
2.2.4.2. *Digit Symbol Coding:* In this subtest, participants are tasked with rapidly transcribing simple symbols. This is facilitated by a reference key that pairs specific numbers with their corresponding symbols. Analogous to the Symbol Search, a timeframe of 120 seconds is dispensed for task completion. This endeavor gauges visuomotor processing speed, short-term visual recall, cognitive flexibility, and sustained attention. This task requires the inhibition of automatic responses to select the correct symbol, testing attention and processing speed (Rosas et al., 2014; Ramos-Henderson et al., 2022; Arango-Lasprilla et al., 2015; Kaufman & Lichtenberger, 1999, 2006; Groth-Marnat, 2009; Lichtenberger & Kaufman, 2009; Sattler & Ryan, 2009).

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

Procedure

This study was conducted within a first-year engineering calculus course at Universidad Católica del Norte. The course curriculum covers elements of real numbers, conics, functions, limits and continuity, derivatives, and their applications. A semester spans 16 weeks, and the course is assigned 5 credits in the Credit Transfer System (CTS). Students are expected to dedicate 7.5 hours per week to the course: 4.5 hours to lectures, 1.5 hours to tutorials, and 1.5 hours to independent study. Assessment is based on three individual tests and group workshops.

The course serves approximately 550 students from various engineering programs, divided into 10 sections of about 55 students each, with one instructor per section. The curriculum and assessments are standardized across all sections. For this study, two sections were selected: one experimental section implementing a student-centered approach, and one control section following a traditional methodology.

During the first week of the course, a subsample of participants from both groups completed selected subtests of the WAIS-IV as a pretest measure. The same assessments were administered at the end of the semester (retest). Examiners—third- and fourth-year psychology students trained by a neuropsychology specialist—were blinded to group assignments.

Traditional and Student-centered approaches.

Traditional Methodology: In the traditional approach, classes consist of lectures reviewing concepts and worked examples. Exercise classes use standardized guides prepared for all sections, and group workshops involve solving a set of problems and submitting the solutions for evaluation.
Example problem:

Determine the derivatives of the following functions:

1. $y = sin(x^2 e^x)$
2. $y = tan\left(\frac{log\,(\,log\,(x^3))}{x^2}\right)$

This task evaluates the student's knowledge of derivative properties and reflects the algebraization of calculus, where the concept is applied in specific contexts.

Student-centered Methodologies: The student-centered approach begins with a motivational session illustrating real-world applications of calculus and emphasizing its importance. Sessions are structured as follows:

1. **Concept review via Kahoot** (15 min), encouraging pre-reading and providing immediate feedback.
2. **Presentation of a contextualized problem** (5 min) related to the topic under review.
3. **Development of theoretical content** (40 min), including theorems, axioms, and examples.
4. **Solution of the contextualized problem** (15 min).
5. **Closing segment** (15 min) summarizing the lesson, previewing the next class, and addressing questions.

Example contextualized problem: "A 7-inch arm connects a piston to a 3-inch radius connecting rod, which rotates counterclockwise at a constant rate of 200 revolutions per minute. Determine the speed of the piston when the angle is 60 degrees." This problem requires integrating motion, measurement, and geometry, applying the derivative concept to related rates.

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

The student-centered section also incorporated Just in Time Teaching (JiTT) (Novak et al., 1999; Patterson et al., 2003), Problem-Based Learning (PBL), formative assessment through Kahoot, and collaborative work. The instructor performed ongoing formative assessments to monitor learning, provide feedback, and adjust instruction. The specific teaching strategies applied in each group are summarized in **Table 2**, highlighting the additional active learning methods implemented in the experimental section compared to the control section.

Table 2.
Comparative teaching strategies in the control and experimental groups

Strategy	Control Group	Experimental Group
Interactive lecture	✓	✓
Problem-solving techniques	✓	✓
Individual problem-solving in class	✓	✓
Just in Time Teaching (JiTT)		✓
Problem-Based Learning (PBL)		✓
Formative assessment (Kahoot)		✓
Collaborative work		✓

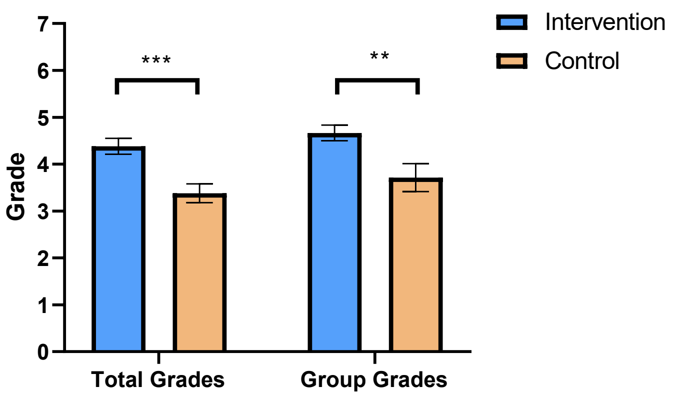
Data Analysis

Cognitive data scores were analyzed with GraphPad Prism version 8 for Windows (GraphPad Software, La Jolla, California, USA, www.graphpad.com). Initially, a Mann Whitney U-test was used to evaluate differences between the groups by comparing their overall course performance. Following this, a Two-Way ANOVA, considering an Alpha of .05, with the Sidak method for multiple comparison correction was conducted. This analysis facilitated intra-group and inter-group comparisons of cognitive performance within the subsample groups. The aim was to ascertain the impact of the intervention on both calculus performance and associated learning outcomes, as well as to gauge cognitive enhancement

RESULTS

First, we compared the overall performance displayed at the end of the course, where participants applied their calculus learning. Notably, the intervention group achieved higher grades than the control group. This trend was consistent when comparing the subsamples from each group. (See Figure 1, **Table 3**).

Figure 1
Comparison of Total and Group Grades



On the left, the grades for the total groups are depicted, with the blue bar representing the intervention group and the pastel peach bar representing the control group. On the right, grades for the subsamples from each group are shown. The performance displayed at the end of the course indicates that the intervention group, represented in blue, consistently achieved higher grades than the control group in both total and subsample comparisons. Bars indicate Standard Error of the Mean (SEM).

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

Table 3.

Calculus Exam Grade Comparisons: Intervention vs. Control Groups

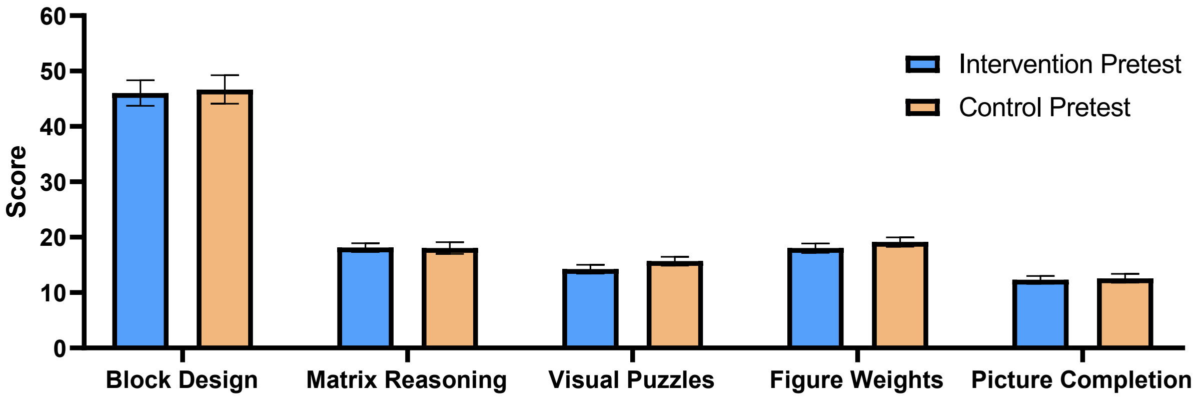
Groups	Grades			Mann Whitney U-test	
	mean	SD	SEM	U	p-value
Intervention Group	4.38	1.07	0.17		
Control Group	3.38	1.36	0.20	467.5	0.0001***
Intervention subsample	4.67	0.89	0.17		
Control subsample	3.71	1.39	0.30	167.5	0.0034**

SD = Standard Deviation; SEM = Standard Error of the Mean.

Second, we performed analyses to evaluate baseline intergroup differences in the pretest cognitive assessment between the control and intervention groups. As expected, there were no significant differences in cognitive measurement scores between the two groups, confirming a consistent baseline for subsequent comparisons (See Figure 2, **Table 4**, Supplementary Table S2).

Figure 2

Pretest intergroup comparisons



The figure shows the performance in perceptual reasoning index subtests. Blue bars represent the intervention group while pastel peach bars represent the control group. In this pretest cognitive assessment, there were no differences between the two groups in their scores, confirming a consistent baseline for further evaluations. Bars indicate Standard Error of the Mean (SEM).

Table 4.

Cognitive Assessment Pretest comparisons between Subsample Groups

Subtests	Intervention Score		Control Score		Sidak's multiple comparisons test		
	mean	SD	mean	SD	Mean rank diff	95% CI of diff.	Adjusted p-value
Block Design	46.03	12.45	46.68	12.12	-0.65	-6.69 to 5.39	>0.9999
Matrix Reasoning	18.14	4.27	18.05	4.87	0.09	-5.95 to 6.13	>0.9999
Visual Puzzles	14.24	4.21	15.68	3.80	-1.44	-7.48 to 4.60	0.9991
Figure Weights	18.03	4.44	19.14	4.02	-1.11	-7.15 to 4.93	>0.9999
Picture Completion	12.31	3.83	12.23	3.87	-0.28	-6.32 to 5.76	0.9999
Digit Span	23.21	3.52	23.41	4.24	-0.20	-6.24 to 5.84	0.9991
Letter-Number Sequencing	17.55	3.67	17.14	2.97	0.41	-5.63 to 6.45	>0.9999
Arithmetic	12.17	3.85	11.64	3.27	0.53	-5.51 to 6.57	>0.9999
Symbol Search	31.97	8.20	30.82	9.32	1.15	-4.89 to 7.19	0.9999
Digit Symbol Coding	67.76	14.89	68.59	16.36	-0.83	-6.87 to 5.21	0.9996

SD = Standard Deviation

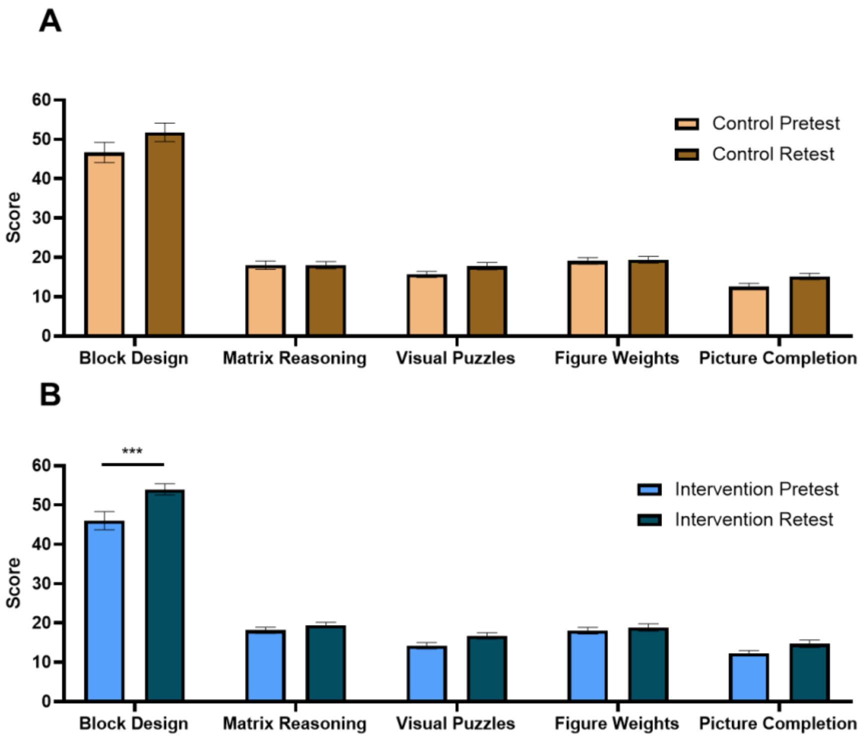
ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

To determine whether the control group and the intervention group exhibited changes in cognitive abilities—specifically in perceptual reasoning, working memory, and processing speed—after completing the calculus course, intragroup analyses were conducted. The control group did not display significant differences in the assessed cognitive abilities when comparing scores from the pretest to the retest (**Table 5**, Supplementary Table S3, Figure 3A). However, the intervention group demonstrated significant improvements in perceptual reasoning, particularly an enhanced performance in the block design subtest (**Table 6**, Supplementary Table S4, Figure 3B).

Table 5.
Cognitive Assessment Pretest-Retest Control Intragroup comparisons

Subtests	Sidak's multiple comparisons test Pretest vs Retest Control Group		
	Mean rank diff	95% CI of diff.	Adjusted p-value
Block Design	-5.14	-11.74 to 1.46	0.2545
Matrix Reasoning	0.00	-6.60 to 6.60	>0.9999
Visual Puzzles	-2.14	-8.74 to 4.46	0.9888
Figure Weights	-0.27	-6.87 to 6.33	>0.9999
Picture Completion	-2.55	-9.15 to 4.05	0.9612
Digit Span	-0.50	-7.10 to 6.10	>0.9999
Letter-Number Sequencing	-0.91	-7.51 to 5.69	>0.9999
Arithmetic	-0.95	-7.55 to 5.65	>0.9999
Symbol Search	-3.36	-9.96 to 3.24	0.8092
Digit Symbol Coding	-6.14	-12.74 to 0.46	0.0879

Figure 3
Intragroup Comparison of Cognitive Abilities after Completing the Calculus Course.



The intragroup comparison for the control group is presented (A). Pastel peach color represents scores from the pretest and brown color represents the retest scores. The control group does not exhibit any significant variations in their assessed cognitive abilities. On the other hand, figure (B) illustrates the intragroup comparison for the intervention group. The pretest scores are in blue, while the retest scores are depicted in dark green. The data indicates that the intervention group experienced significant improvements, especially in block design subtest. Bars indicate Standard Error of the Mean (SEM).

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

Table 6.

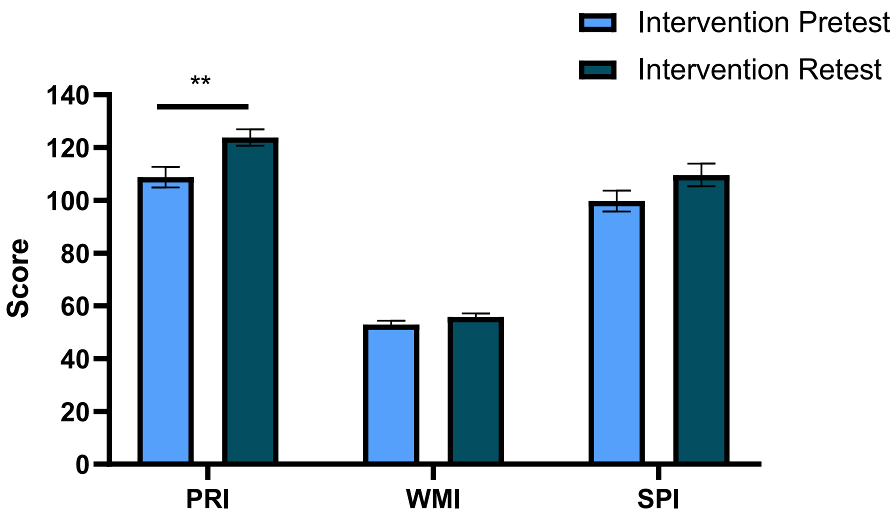
Cognitive Assessment Pretest-Retest Intervention Intragroup comparisons

Subtests	Sidak's multiple comparisons test Pretest vs Retest Intervention Group		
	Mean rank diff	95% CI of diff.	Adjusted p-value
Block Design	-8.00	-13.42 to -2.58	0.0004***
Matrix Reasoning	-1.310	-6.73 to 4.11	0.9990
Visual Puzzles	-2.480	-7.90 to 2.94	0.8914
Figure Weights	-0.8300	-6.25 to 4.59	>0.9999
Picture Completion	-2.410	-7.83 to 3.01	0.9078
Digit Span	-1.650	-7.07 to 3.77	0.9932
Letter-Number Sequencing	-0.9000	-6.32 to 4.52	>0.9999
Arithmetic	-0.3800	-5.80 to 5.04	>0.9999
Symbol Search	-4.740	-10.16 to 0.68	0.1342
Digit Symbol Coding	-5.030	-10.45 to 0.39	0.0898

When the total scores of the indexes—including the Perceptual Reasoning Index, Working Memory Index, and the Speed Processing Index—were compared, the results further elucidated the cognitive changes following the completion of the calculus course. While individual subtests provided insights into specific cognitive abilities, the consolidated scores (or indexes) offered a broader perspective on overall cognitive skills. Notably, only the intervention group exhibited significant differences in their overall perceptual reasoning abilities (Figure 4). In contrast, no significant differences were observed in the pre-retest index scores for the control group (**Table 7**, Supplementary Table S5 and Table S6).

Figure 4.

Intragroup Comparison of Cognitive Abilities Indexes



The figure shows the intragroup comparison for the intervention group. The pretest scores are in blue, while the retest scores are depicted in dark green. The data indicates that the intervention group experienced significant improvements in the Perceptual Reasoning Index. Bars indicate Standard Error of the Mean (SEM). PRI: Perceptual Reasoning Index; WMI: Working Memory Index; SPI: Speed Processing Index.

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

Table 7.
Cognitive Assessment Pretest-Retest comparisons intragroup for the total indexes

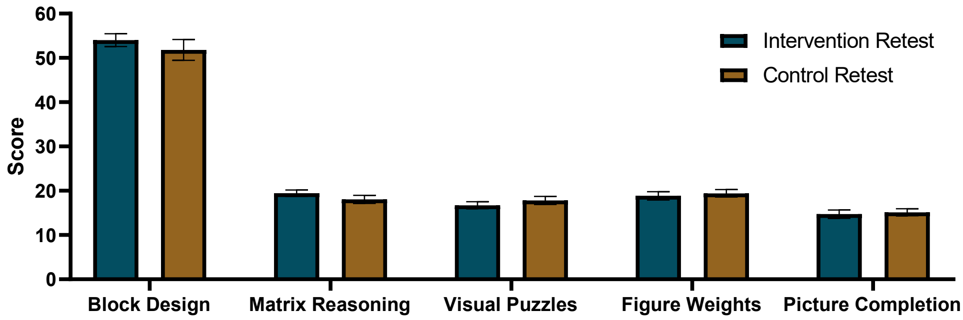
Indexes	Intervention Score Pretest		Intervention Score Retest		Sidak's multiple comparisons test		
	mean	SD	mean	SD	Mean rank diff	95% CI of diff.	Adjusted p-value
Perceptual Reasoning	108.8	21.08	123.8	16.84	-15.00	-26.01 to -3.97	0.0037**
Working Memory	52.93	7.91	55.86	6.68	-2.930	-13.94 to 8.08	0.8908
Speed Processing	99.72	21.16	109.6	23.05	-9.880	-20.89 to 1.13	0.0927

Indexes	Control Score Pretest		Control Score Retest		Sidak's multiple comparisons test		
	mean	SD	mean	SD	Mean rank diff	95% CI of diff.	Adjusted p-value
Perceptual Reasoning	111.8	20.85	122.2	19.96	-10.40	-23.59 to 2.789	0.1658
Working Memory	52.18	7.255	54.55	8.388	-2.370	-15.56 to 10.82	0.9622
Speed Processing	99.41	22.67	108.9	22.15	-9.490	-22.68 to 3.70	0.2317

SD = Standard Deviation

In our subsequent assessment of the intergroup retest scores between the control and intervention groups, statistical analyses indicated nonsignificant differences across all metrics, including the block design subtest. This outcome was incongruent with our preliminary expectations (**Table 8** and Supplementary Table S7, Figure 5).

Figure 5.
Retest intergroup comparisons



The figure shows the performance in perceptual reasoning index subtests. Dark green bars represent the intervention group while brown bars represent the control group. In this retest cognitive assessment, there were no differences between the two groups in their scores. Bars indicate Standard Error of the Mean (SEM).

Table 8.
Cognitive Assessment Retest comparisons between Subsample Groups

Subtests	Intervention Score		Control Score		Sidak's multiple comparisons test		
	mean	SD	mean	SD	Mean rank diff	95% CI of diff.	Adjusted p-value
Block Design	54.03	7.81	51.82	11.07	2.21	-3.72 to 8.14	0.9700
Matrix Reasoning	19.45	3.996	18.05	4.370	1.40	-4.54 to 7.34	0.9992
Visual Puzzles	16.72	4.284	17.82	4.360	-1.10	-7.04 to 4.84	>0.9999
Figure Weights	18.86	5.041	19.41	3.912	-0.55	-6.49 to 5.39	>0.9999
Picture Completion	14.72	5.112	15.14	3.733	-0.42	-6.36 to 5.52	>0.9999
Digit Span	24.86	3.16	23.91	4.450	0.95	-4.99 to 6.89	>0.9999
Letter-Number Sequencing	18.45	2.971	18.05	2.968	0.40	-5.54 to 6.34	>0.9999
Arithmetic	12.55	3.699	12.59	4.067	-0.04	-5.98 to 5.90	>0.9999
Symbol Search	36.76	7.448	34.18	8.204	2.53	-3.41 to 8.47	0.9280
Digit Symbol Coding	72.79	16.95	74.73	17.32	-1.94	-7.88 to 4.00	0.9882

SD = Standard Deviation

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

DISCUSSION

The outcomes of this study provide relevant insights into the relationship between pedagogical methods in calculus instruction and the enhancement of specific cognitive abilities. The intervention group's superior performance in calculus, as evidenced by higher grades compared to the control group, highlights the potential efficacy of a student-centered teaching approach. This finding is consistent with prior literature supporting the use of applied, real-world, and practical teaching methods in STEM disciplines (Artigue, 1998; Camarena, 2010).

It is plausible that this strategy facilitated a deeper understanding of calculus concepts and their applications, translating into improved academic performance. A major challenge in teaching calculus is avoiding its reduction to mechanical procedures without comprehension of underlying meaning. In our study, this was addressed through contextualized problems designed around real-life engineering scenarios, giving greater meaning to calculus concepts and promoting meaningful learning. By contextualized problems, we refer to applying mathematical concepts to authentic engineering problems, using the language and notation of the field (Kukliansky & Rozenes, 2015). This contrasts with traditional methods focused on abstract exercises disconnected from real-world contexts. Gerofsky (2004) emphasizes that word problems can function as narratives reflecting cultural and social contexts, enhancing their relevance for students.

The methodology implemented in this study aligns with student-centered approaches. By assigning an active role to students through debates, problem-solving, and reflective processes, it fosters procedural fluency, strong theoretical understanding, and better course performance (Stanberry & Payne, 2023). Based on these approaches, several strategies emerge as effective for enhancing calculus learning and cognitive development: active participation in class (Freeman et al., 2014), connecting problems to real-world applications (Mazzeo et al., 2003; Johnson, 2002), use of educational technology for visualizing abstract concepts (Dockendorff & Zaccarelli, 2024), scaffolded practice (Holton & Clarke, 2006), collaboration and team-based learning (Burgess et al., 2020; Naughton et al., 2020), pre-class preparation (Erbil, 2020; Cevikbas & Kaiser, 2020; Strelan et al., 2020), regular review and spaced repetition (Roediger & Butler, 2011), and goal setting with effective time management (Zimmerman, 2002). These strategies, aligned with the pedagogical framework of this study, provide a comprehensive basis for improving performance in calculus. Furthermore, the implemented method creates a context that can foster other learning variables such as motivation, imagination, language skills, teamwork, and representation skills, improving preparation for subsequent calculus courses.

Regarding cognitive outcomes, both groups were similar at baseline, eliminating initial differences as a confounding factor. After instruction, the control group’s cognitive scores remained stable, while the intervention group improved in perceptual reasoning, specifically in the Block Design subtest. This suggests that the student-centered approach may promote visuospatial reasoning skills, crucial for representing calculus concepts—such as interpreting or sketching graphs, understanding curve behavior, and grasping geometric perspectives of integration and differentiation (Battista, 2007; Sorby et al., 2013). For example, stronger perceptual reasoning could aid in visualizing Riemann sums to approximate areas under a curve or in understanding solids of revolution using methods like cylinders and rings.

Despite these intragroup improvements, no significant intergroup differences were found in post-test cognitive scores. Possible explanations include the broad nature of WAIS measures, which may dilute specific skill gains, and potential ceiling effects if participants were already near their maximum capacity. Another relevant consideration is the well-documented difficulty of achieving far transfer in cognitive training. Far transfer refers to the generalization of improvements from a specific trained domain to broader cognitive abilities measured through standardized assessments. Research in cognitive psychology has consistently shown that such transfer is difficult to achieve, particularly when the intervention is relatively short in duration (Barnett & Ceci, 2002; Sala & Gobet, 2020; Simons et al., 2016). In this context, a single semester of calculus instruction—even when delivered through a student-centered methodology—may promote improvements in task-relevant skills such as visuospatial reasoning within mathematical contexts, yet these gains may not immediately translate into measurable differences between groups on broad psychometric indicators of fluid cognitive ability. Future studies with longer interventions or repeated exposure to cognitively demanding learning environments may help clarify whether such pedagogical approaches can produce more robust far-transfer effects.

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

A limitation of the present study concerns the reduced sample size used for the cognitive analyses. Although the overall sample included 85 students, WAIS-IV assessments were administered to a subsample (29 in the intervention group and 22 in the control group) due to the time-intensive nature of the testing procedure. A sensitivity analysis indicated that the available sample size was sufficient to detect medium-sized effects, but smaller cognitive changes may have gone undetected. Future studies with larger samples would allow a more precise estimation of the cognitive effects associated with student-centered calculus instruction.

Our findings indicate that a student-centered approach can positively influence both academic performance and perceptual reasoning. While it is intuitive that real-world applications enhance learning (Bulte et al., 2006), this study underscores the importance of validating such approaches through empirical evidence (Bulte et al., 2006; Deslauriers et al., 2019). The results highlight the value of purposeful teaching methods in improving academic and cognitive outcomes.

CONCLUSIONS

This study demonstrates that a student-centered approach can enhance both calculus performance and specific cognitive abilities, particularly visuospatial reasoning. While the cognitive effects observed were modest, the results underscore the importance of integrating contextualized, interactive, and problem-based methods in STEM education. Such approaches not only improve academic achievement but may also support the development of cognitive skills fundamental to success in advanced mathematics and engineering.

Future studies should further investigate the mechanisms through which pedagogical strategies influence cognitive functions, explore scalability across diverse educational contexts, and assess long-term retention. Ultimately, aligning calculus instruction with student-centered principles represents a promising pathway for improving both academic and cognitive outcomes in higher education.

ACKNOWLEDGMENTS

The authors extend their gratitude to the students of the Pedagogy in Mathematics program at Universidad Católica del Norte who developed a set of situational problems for use in the calculus course: Vicente Alfaro, Felipe Sanchez, Mauro Olivarez, Johiner Palominos, Jose Vergara, and Maximiliano Ossandon. Their contributions were crucial to the enrichment of this work.

Additionally, profound gratitude is expressed to the students from the Psychology Program who participated in the FDPD-UCN project "*Implementación de un método de enseñanza del cálculo y su efecto en el desarrollo de habilidades cognitivas*" at Universidad Católica del Norte, for their invaluable feedback and assistance with data collection: Carolayn Escudero, Aracely Oliva, Camila Aguilar, Camila Espinoza, Catalina Zúñiga, María Ávalos, Natalia Acuña, Jennifer Burgos, and Paulina Aliaga. Their insights and diligence greatly enhanced the quality of this research.

This study was financially supported by the DGPRE of the Universidad Católica del Norte.

Declaration of Interest Statement

The authors declare that they have no conflicts of interest.

Ethical Statement

This study involved human participants and was approved by the Ethics Committee of the Universidad Católica del Norte, Antofagasta, Chile. ID reference number: 038/2022. Participants gave informed consent to participate in the study before taking part.

Data availability statement

Data is available upon reasonable request.

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

Author Contributions.

RJ: Conceptualization, Resources, Methodology, Investigation, Writing - original draft, Writing - review & editing, Supervision, Project administration, Funding acquisition. **SA-C:** Conceptualization, Writing - original draft, Writing - review & editing, Funding acquisition. **WT:** Conceptualization, Writing - original draft, Writing - review & editing, Funding acquisition. **EH:** Conceptualization, Investigation, Data curation, Writing - original draft, Writing - review & editing. **NH:** Investigation, Data curation, Writing - review & editing. **LF:** Investigation, Data curation, Writing - review & editing. **MD-S:** Conceptualization, Resources, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing - original draft, Writing - review & editing, Supervision, Visualization, Funding acquisition.

REFERENCIAS

Agencia de Calidad de la Educación. (2022). Informe nacional PISA 2022: Evaluación internacional de estudiantes tras la pandemia.

Arango-Lasprilla, J., Rivera, D., Rodríguez, G., Garza, M., Galarza-Del-Angel, J., Rodríguez, W., Velázquez-Cardoso, J., Aguayo, A., Schebela, S., Weil, C., Longoni, M., Aliaga, A., Ocampo-Barba, N., Saracho, C., Panyavin, I., Esenarro, L., Martínez, C., De la Cadena, C. G., & Perrin, P. (2015). Symbol Digit Modalities Test: Normative data for the Latin American Spanish speaking adult population. Neurorehabilitation, 37(4), 625-638. https://doi.org/10.3233/nre-151282

Aarnoudse-Moens, C. S., Weisglas-Kuperus, N., Duivenvoorden, H. J., van Goudoever, J. B., & Oosterlaan, J. (2013). Executive function and IQ predict mathematical and attention problems in very preterm children. PloS one, 8(2), e55994. https://doi.org/10.1371/journal.pone.0055994

Artigue, M. (1998). Enseñanza y aprendizaje del análisis elemental: ¿qué se puede aprender de las investigaciones didácticas y los cambios curriculares. Revista Latinoamericana de Investigación en Matemática Educativa, 1(1), 40-55

Barnett, S. M., & Ceci, S. J. (2002). When and where do we apply what we learn? A taxonomy for far transfer. Psychological Bulletin, 128(4), 612–637. https://doi.org/10.1037/0033-2909.128.4.612

Battista, M. T. (2007). The development of geometric and spatial thinking. In F. K. Lester (Ed.), Second handbook of research on mathematics teaching and learning (pp. 843-908). Information Age.

Battista, M. T. (2017). Reasoning and sense making in the elementary grades. National Council of Teachers of Mathematics.

Black, P., & Wiliam, D. (2009). Developing the theory of formative assessment. Educational Assessment, Evaluation and Accountability, 21(1), 5–31. https://doi.org/10.1007/s11092-008-9068-5

Berkowitz, M., Edelsbrunner, P. A., & Stern, E. (2022). The relation between working memory and mathematics performance among students in math-intensive STEM programs. Intelligence, 92, 101649. https://doi.org/10.1016/j.intell.2022.101649

Bressoud, D. M., Mesa, V., & Rasmussen, C. (2015). Insights and recommendations from the MAA national study of college calculus. Mathematics Association of America.

Bonwell, C. C., & Eison, J. A. (1991). Active learning: Creating excitement in the classroom. ASHE-Eric Higher Education Report No. 1. George Washington University.

Bozzi, M., Raffaghelli, J. E., & Zani, M. (2021). Peer learning as a key component of an integrated teaching method: Overcoming the complexities of physics teaching in large size classes. Education Sciences, 11(2), Article 67. https://doi.org/10.3390/educsci11020067

Bulte, A. M. W., Westbroek, H. B., de Jong, O., & Pilot, A. (2006). A research approach to designing chemistry education using authentic practices as contexts. International Journal of Science Education, 28(9), 1063-1086. https://doi.org/10.1080/09500690600702520

Burgess, A., van Diggele, C., Roberts, C., & Mellis, C. (2020). Team-based learning: design, facilitation and participation. BMC medical education, 20(Suppl 2), 461. https://doi.org/10.1186/s12909-020-02287-y

Camacho-Machín, M., & Guerrero-Ortiz, C. (2015). Identifying and exploring relationships between contextual situations and ordinary differential equations. International Journal of Mathematical Education in Science and Technology, 46(8), 1077–1095. https://doi.org/10.1080/0020739X.2015.1025877

Camarena, P. (2010). Aportaciones de Investigación al aprendizaje y enseñanza de la matemática en Ingeniería. Ingeniería en Comunicaciones y Electrónica. IPN.

Carroll, J. B. (1993). Human cognitive abilities: a survey of factor-analytic Studies. Cambridge University Press.

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

Caviola, S., Mammarella, I. C., Cornoldi, C., & Lucangeli, D. (2012). The involvement of working memory in children's exact and approximate mental addition. Journal of experimental child psychology, 112(2), 141–160. https://doi.org/10.1016/j.jecp.2012.02.005

Cevikbas, M., & Kaiser, G. (2020). Flipped classroom as a reform-oriented approach to teaching mathematics. ZDM: the international journal on mathematics education, 52(7), 1291–1305. https://doi.org/10.1007/s11858-020-01191-5

Cheng, D., Shi, K., Wang, N., Miao, X., & Zhou, X. (2021). Examining the Differential Role of General and Specific Processing Speed in Predicting Mathematical Achievement in Junior High School. Journal of Intelligence, 10(1), 1. https://doi.org/10.3390/jintelligence10010001

Cheung, A. C. K., & Slavin, R. E. (2013). The Effectiveness of Educational Technology Applications for Enhancing Mathematics Achievement in K-12 Classrooms: A Meta-Analysis. Educational Research Review, 9, 88-113. https://doi.org/10.1016/j.edurev.2013.01.001

Collins, A., & Koechlin, E. (2012). Reasoning, learning, and creativity: Frontal lobe function and human decision-making. PLoS Biology, 10(3). https://doi.org/10.1371/journal.pbio.1001293

Cordero Osorio, F., & Flores Estrella, R. B. (2007). El uso de las gráficas en el discurso matemático escolar. Un estudio socioepistemológico en el nivel básico a través de los libros de texto. Revista Latinoamericana de Investigación en Matemática Educativa, RELIME, 10(1), 7-38.

Cragg, L., Richardson, S., Hubber, P. J., Keeble, S., & Gilmore, C. (2017). When is working memory important for arithmetic? The impact of strategy and age. PloS one, 12(12), e0188693. https://doi.org/10.1371/journal.pone.0188693

Davidson, M. C., Amso, D., Anderson, L. C., & Diamond, A. (2006). Development of cognitive control and executive functions from 4 to 13 years: Evidence from manipulations of memory, inhibition, and task switching. Neuropsychologia, 44(11), 2037–2078. https://doi.org/10.1016/j.neuropsychologia.2006.02.006

DeBaggis, H. & Miller, K. (1966). Foundations of the Calculus. Saunders.

De Corte, E. & Verschaffel, L. (2008). Apprendre et enseigner les mathématiques : un cadre conceptuel pour concevoir des environnements d'enseignement-apprentissage stimulants. Que disent les recherches psychopédagogiques. In M. Crahay, L. Verschaffel, E. De Corte, & J. Grégoire (Eds.), Enseignement et apprentissage des mathématiques: Que disent les recherches psychopédagogiques? (pp. 25-54). Brussels: De Boeck Université. https://doi.org/10.3917/dbu.craha.2008.01.0025

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of mathematical thinking: behavioral and brain-imaging evidence. Science (New York, N.Y.), 284(5416), 970–974. https://doi.org/10.1126/science.284.5416.97

Departamento de Evaluación, Medición y Registro Educacional (2024) Informe de resultados PAES Regular Admisión 2024. Departamento de Evaluación, Medición y Registro Educacional. https://www.mifuturo.cl/reportes-en-materia-de-educacion-superior/

Deslauriers, L., McCarty, L. S., Miller, K., Callaghan, K., & Kestin, G. (2019). Measuring actual learning versus feeling of learning in response to being actively engaged in the classroom. Proceedings of the National Academy of Sciences, 116(39), 19251-19257. https://doi.org/10.1073/pnas.1821936116

Diamond, A. (2013). Executive functions. Annual review of psychology, 64, 135–168. https://doi.org/10.1146/annurev-psych-113011-143750

Dockendorff, M., & Zaccarelli, F. G. (2024). Successfully preparing future mathematics teachers for digital technology integration: a literature review. International Journal of Mathematical Education in Science and Technology, 1–32. https://doi.org/10.1080/0020739X.2024.2309273

Domic-Siede, M., Irani, M., Ramos-Henderson, M., Calderón, C., Ossandón, T., & Perrone-Bertolotti, M. (2022). La planificación cognitiva en el contexto de la evaluación neuropsicológica e investigación en neurociencia cognitiva: una revisión sistemática. Terapia Psicológica, 40(3), 367-395. https://doi.org/10.4067/S0718-48082022000300367

Domic-Siede, M., Irani, M., Valdés, J., Perrone-Bertolotti, M., & Ossandón, T. (2021). Theta activity from frontopolar cortex, mid-cingulate cortex and anterior cingulate cortex shows different roles in cognitive planning performance. NeuroImage, 226, 117557. https://doi.org/10.1016/j.neuroimage.2020.117557

Domic-Siede, M., Irani, M., Valdés, J., Rodríguez, M., Follet, B., Perrone-Bertolotti, M., & Ossandón, T. (2023). A Visuospatial Planning Task Coupled with Eye-Tracker and Electroencephalogram Systems. Journal of visualized experiments: JoVE, (193), 10.3791/64622. https://doi.org/10.3791/64622

Domic-Siede, M., Ortiz, R., Ávalos, M., Salazar, N., Burgos, J., Rosales, C., Ramos-Henderson, M., Véliz-García, O. & Calderón, C. (2024). Executive Functions and their Relationship with Age: Insights from a Novel Neuropsychological Assessment Battery in Children - A Pilot Study. Applied neuropsychology Child. https://doi.org/10.1080/21622965.2024.2381199.

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

Dresel, M., Daumiller, M., Spear, J., Janke, S., Dickhäuser, O., & Steuer, G. (2024). Learning from errors in mathematics classrooms: Development over 2 years in dependence of perceived error climate. The British journal of educational psychology, 10.1111/bjep.12697. Advance online publication. https://doi.org/10.1111/bjep.12697

Duran, P. A., Castillo, A. J., Watson, C., Fuller, E., Potvin, G., & Kramer, L. H. (2022). Student attitudes and achievement in active learning calculus. International Journal of Mathematical Education in Science and Technology, 55(3), 759–780. https://doi.org/10.1080/0020739X.2022.2150902

Ellis, B. M., Larsen, S., Voigt, M., & Vroom, K. (2021). Where Calculus and Engineering Converge: an Analysis of Curricular Change in Calculus for Engineers. International Journal of Research in Undergraduate Mathematics Education, 7(2), 379–399. https://doi.org/10.1007/s40753-020-00130-9

Erbil D. G. (2020). A Review of Flipped Classroom and Cooperative Learning Method Within the Context of Vygotsky Theory. Frontiers in psychology, 11, 1157. https://doi.org/10.3389/fpsyg.2020.01157

Evans, J., Over, D.E. Manktelow, K.I. (1993). Reasoning, decision making and rationality. Cognition, 49(1-2), 165–187. https://doi.org/10.1016/0010-0277(93)90039-x

Faul, F., Erdfelder, E., Lang, A. G., & Buchner, A. (2007). G*Power 3: a flexible statistical power analysis program for the social, behavioral, and biomedical sciences. Behavior research methods, 39(2), 175–191. https://doi.org/10.3758/bf03193146

Fraihat, M. a. K., Khasawneh, A. A., & Al-Barakat, A. A. (2022). The effect of situated learning environment in enhancing mathematical reasoning and proof among tenth grade students. Eurasia Journal of Mathematics, Science and Technology Education, 18(6), em2120. https://doi.org/10.29333/ejmste/12088

Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. Proceedings of the National Academy of Sciences, 111(23), 8410–8415. https://doi.org/10.1073/pnas.1319030111

García Retana, J. Á., (2013). La problemática de la enseñanza y el aprendizaje del cálculo para ingeniería. Revista Educación, 37(1), 29-42.

Geary, D. C., & Hoard, M. K. (2005). Learning disabilities in arithmetic and mathematics: Theoretical and empirical perspectives. In J. I. D. Campbell (Ed.), Handbook of mathematical cognition (pp. 253–267). Psychology Press.

Gerofsky, S. (2004). A man left Albuquerque heading east: Word problems as genre in mathematics education. Peter Lang Publishing.

Green, C. T., Bunge, S. A., Briones Chiongbian, V., Barrow, M., & Ferrer, E. (2017). Fluid reasoning predicts future mathematical performance among children and adolescents. *Journal of experimental child psychology*, *157*, 125–143. https://doi.org/10.1016/j.jecp.2016.12.005

Groth-Marnat, G. (2009). Handbook of psychological assessment (5th ed.). Wiley.

Hagman, J. E., Johnson, E., & Fosdick, B. K. (2017). Factors contributing to students and instructors experiencing a lack of time in college calculus. International Journal of STEM Education, 4(1). https://doi.org/10.1186/s40594-017-0070-7

Hayes-Roth, B., & Hayes-Roth, F. (1979). A cognitive model of planning. *Cognitive Science,* *3*(4), 275–310. https://doi.org/10.1207/s15516709cog0304_1

Hmelo-Silver, C. E. (2004). Problem-Based Learning: What and How Do Students Learn? *Educational Psychology Review, 16*(3), 235-266. https://doi.org/10.1023/B:EDPR.0000034022.16470.f3

Holton, D., & Clarke, D. (2006). Scaffolding and metacognition. *International Journal of Mathematical Education in Science and Technology, 37*(2), 127–143. https://doi.org/10.1080/00207390500285818

Hughes-Hallett, D., Gleason, A. M., & McCallum, W. G. (2020). *Calculus: Single and Multivariable*. (8th ed). John Wiley & Sons.

Johnson, E. B. (2002). Contextual teaching and learning: What it is and why it's here to stay. Corwin Press.

Karunakaran, S. S. & Higgins, A. (Eds.). (2021). Research in Undergraduate Mathematics Education Reports.

Kaufman, A. S., & Lichtenberger, E. O. (1999). Essentials of WAIS-III assessment. Wiley.

Kaufman, A. S., & Lichtenberger, E. O. (2006). *Assessing Adolescent and Adult Intelligence* (3rd ed.). Wiley.

Keiler L. S. (2018). Teachers' roles and identities in student-centered classrooms. *International journal of STEM education, 5*(1), 34. https://doi.org/10.1186/s40594-018-0131-6

Keogh, B. K. (1994). A matrix of decision points in the measurement of learning disabilities. In G. R. Lyon (Ed.), *Frames of reference for the assessment of learning disabilities* (pp. 15–26). Brookes.

Kozma, R. B. (2003). Technology and Classroom Practices: An International Study. *Journal of Research on Technology in Education, 36*(1), 1-14. https://doi.org/10.1080/15391523.2003.10782399

Kreyszig, E. (2011). *Advanced Engineering Mathematics* (10th ed). John Wiley & Sons.

Kukliansky, I., & Rozenes, S. (2015). The contextual learning approach in engineering education. In *1st International Conference on Higher Education Advances (HEAd'15)*. Universitat Politècnica de València. https://doi.org/10.4995/HEAd15.2015.280

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

Lam, T. T. (2007). Contextual approach in teaching mathematics: an example using the sum of series of positive integers. *International Journal of Mathematical Education in Science and Technology, 38*(2), 273–282. https://doi.org/10.1080/00207390600913376

Lezak, M. D. (1995). Neuropsychological assessment (3rd ed.). Oxford University Press.

Lichtenberger, E. O., & Kaufman, A. S. (2009). *Essentials of WAIS-IV Assessment*. Wiley.

Lugosi, E., & Uribe, G. (2020). Active learning strategies with positive effects on students' achievements in undergraduate mathematics education. *International Journal of Mathematical Education in Science and Technology, 53*(2), 403–424. https://doi.org/10.1080/0020739X.2020.1773555

Mazzeo, C., Rab, S. Y., Alssid, J. L. (2003). *Building bridges to college and careers: contextualized basic skills programs at community colleges*. Workforce Strategy Center.

McCarthy J.P. & Anderson L. (2000). Active Learning Techniques Versus Traditional Teaching Styles: Two Experiments from History and Political Science. *Innovative Higher Education, 24*(4).https://doi.org/10.1023/B:IHIE.0000047415.48495.05

McLean, J. F., & Hitch, G. J. (1999). Working memory impairments in children with specific arithmetic learning difficulties. *Journal of experimental child psychology, 74*(3), 240–260. https://doi.org/10.1006/jecp.1999.2516

McQuarrie, D. (2003). *Mathematical Methods for Scientists and Engineers*. University Science Books.

Mix, K. S., Levine, S. C., Cheng, Y. L., Young, C., Hambrick, D. Z., Ping, R., & Konstantopoulos, S. (2016). Separate but correlated: The latent structure of space and mathematics across development. *Journal of experimental psychology. General*, *145*(9), 1206–1227. https://doi.org/10.1037/xge0000182

Moreno Bayardo, M. G., (2005). Potenciar la educación. un currículum transversal de formación para la investigación. *REICE. Revista Iberoamericana sobre Calidad, Eficacia y Cambio en Educación, 3*(1), 520-540.

Morris, R. G. & Ward, G. (2005). Introduction to the psychology of planning. In R. Morris & G. Ward (Eds.), *The cognitive psychology of planning* (pp. 199-227). Hove, U.K.: Psychology Press.

Naughton, L., Butler, R., Parkes, A., Wilson, P., & Gascoyne, A. (2020). Raising aspirations using elements of team-based learning in mathematics: a pilot study. *International Journal of Mathematical Education in Science and Technology, 52*(10), 1491–1507. https://doi.org/10.1080/0020739X.2020.1770880

Novak, G. M., Patterson, E. T., Gavrin, A. D., & Christian, W. (1999). Just-in-time teaching: Blending active learning with web technology. Prentice Hall.

Patterson, B.M., Ozbayrac, M., Papadopoulou, T. (2003). Simulation of JiT performance in a printing shop. Proceeding of the winter simulations Conference, 8-10 december 2002, San Diego CA. USA.

Peterson, R. K., Ng, R., Ludwig, N. N., & Jacobson, L. A. (2023). Working Memory and Processing Speed Predict Math Skills in Pediatric Brain Tumor Survivors. *Journal of pediatric hematology/oncology*, *45*(3), e350–e355. https://doi.org/10.1097/MPH.0000000000002561

Pintor, E., Gargantilla, P., Herreros, B., & López, M. (2014). El aprendizaje basado en juegos: experiencias docentes en la aplicación de la plataforma virtual "Kahoot". En *XI Jornadas Internacionales de Innovación Universitaria: Educar para transformar*. Universidad Europea de Madrid.

Prince, M. (2004). Does active learning work? A review of the research. *Journal of Engineering Education, 93*(3), 223-231. https://doi.org/10.1002/j.2168-9830.2004.tb00809.x

amos-Henderson, M., Calderón, C., Toro-Roa, I., Aguilera-Choppelo, R., Palominos, D., Soto-Añari, M., López, N., & Domic-Siede, M. (2022). The cumulative effect of fibromyalgia symptoms on cognitive performance: The mediating role of pain. *Applied neuropsychology. Adult*, 1–11. Advance online publication. https://doi.org/10.1080/23279095.2022.2122828

Reynolds, C. R. (1997). Forward and backward memory span should not be combined for clinical analysis. *Archives of Clinical Neuropsychology, 12*(1), 29–40.

Roediger, H. L., & Butler, A. C. (2011). The critical role of retrieval practice in long-term retention. *Trends in Cognitive Sciences, 15(*1), 20-27. https://doi.org/10.1016/j.tics.2010.09.003

Rosas, R., Tenorio, M., Pizarro, M., Cumsille, P., Bosch, A., Arancibia, S., Carmona-Halty, M., Pérez-Salas, C. P., Pino, E., Vizcarra, B., & Zapata-Sepúlveda, P. (2014). Estandarización de la Escala Wechsler de Inteligencia para Adultos: Cuarta edición en Chile. *Psykhe, 23*(1), 1–18. https://doi.org/10.7764/psykhe.23.1.529

Rourke B. P. (1993). Arithmetic disabilities, specific and otherwise: a neuropsychological perspective. *Journal of learning disabilities, 26*(4), 214–226. https://doi.org/10.1177/002221949302600402

Sala, G., & Gobet, F. (2020). Working memory training in typically developing children: A multilevel meta-analysis. *Psychonomic bulletin & review*, *27*(3), 423–434. https://doi.org/10.3758/s13423-019-01681-y

Sattler, J. M., & Ryan, J. J. (2009). *Assessment with the WAIS-IV*. Author.

Santa-Cruz, C., & Rosas, R. (2017). Mapping of Executive Functions / Cartografía de las Funciones Ejecutivas. *Estudios de Psicología, 38*(2), 284–310. https://doi.org/10.1080/02109395.2017.1311459

ISSN: 0718-4123
DOI: 10.7714/CNPS/20.1.207

Savery, J. R. (2006). Overview of Problem-based Learning: Definitions and Distinctions. *Interdisciplinary Journal of Problem-based Learning, 1*(1), 9-20. https://doi.org/10.7771/1541-5015.1002

Schneider, W. J., & McGrew, K. S. (2012). The Cattell-Horn-Carroll model of intelligence. In D. P. Flanagan & P. L. Harrison (Eds.), *Contemporary intellectual assessment: Theories, tests, and issues* (3rd ed., pp. 99–144). The Guilford Press.

Share, D. L., Moffitt, T. E., & Silva, P. A. (1988). Factors associated with arithmetic-and-reading disability and specific arithmetic disability. *Journal of learning disabilities, 21*(5), 313–320. https://doi.org/10.1177/002221948802100515

Shé, C. N., Mac an Bhaird, C., & Fhloinn, E. N. (2023). Factors that influence student engagement with technology-enhanced resources for formative assessments in first-year undergraduate mathematics. International *Journal of Mathematical Education in Science and Technology*, 1–19. https://doi.org/10.1080/0020739X.2023.2182725

Simons, D. J., Boot, W. R., Charness, N., Gathercole, S. E., Chabris, C. F., Hambrick, D. Z., & Stine-Morrow, E. A. (2016). Do "Brain-Training" Programs Work?. *Psychological science in the public interest: a journal of the American Psychological Society*, *17*(3), 103–186. https://doi.org/10.1177/1529100616661983

Sokolowski, A., Yalvac, B., & Loving, C. (2011). Science modelling in pre-calculus: how to make mathematics problems contextually meaningful. *International Journal of Mathematical Education in Science and Technology, 42*(3), 283–297. https://doi.org/10.1080/0020739X.2010.526255

Sorby, S., Casey, B., Veurink, N., & Dulaney, A. (2013). The role of spatial training in improving spatial and calculus performance in engineering students. *Learning and Individual Differences, 26*, 20–29. https://doi.org/10.1016/j.lindif.2013.03.010

Spooner, K. (2023). Using mathematical modelling to provide students with a contextual learning experience of differential equations. *International Journal of Mathematical Education in Science and Technology, 55*(2), 565–573. https://doi.org/10.1080/0020739X.2023.2244472

Stanberry, M. L., & Payne, W. R. (2023). Teaching Undergraduate Calculus at an Urban HBCU through a Global Pandemic. *International Journal of Education In Mathematics, Science And Technology, 11*(2), 340-357. https://doi.org/10.46328/ijemst.2557

Strelan, P., Osborn, A., & Palmer, E. K. (2020). The flipped classroom: A meta-analysis of effects on student performance across disciplines and education levels. *Educational Research Review, 30*, 100314. https://doi.org/10.1016/j.edurev.2020.100314

Sutherland, T., & Bonwell, C. (1996). The active learning continuum: Choosing activities to engage students in the classroom. In T. Sutherland & C. Bonwell (Eds.), Using active learning in college classes: A range of options for faculty (New Directions for Teaching and Learning, No. 67, pp. 3–17). Jossey-Bass.

Van Bueren, N. E. R., van der Ven, S. H. G., Roelofs, K., Cohen Kadosh, R., & Kroesbergen, E. H. (2022). Predicting Math Ability Using Working Memory, Number Sense, and Neurophysiology in Children and Adults. *Brain sciences*, *12*(5), 550. https://doi.org/10.3390/brainsci12050550

Visu-Petra, L., Stanciu, O., Benga, O., Miclea, M., & Cheie, L. (2014). Longitudinal and concurrent links between memory span, anxiety symptoms, and subsequent executive functioning in young children. *Frontiers in psychology*, *5*, 443. https://doi.org/10.3389/fpsyg.2014.00443

Wechsler, D. (2008). *Wechsler Adult Intelligence Scale* (4th ed.). Pearson.

Zimmerman, B. J. (2002). Becoming a Self-Regulated Learner: An Overview. *Theory Into Practice, 41*(2), 64–70. https://doi.org/10.1207/s15430421tip4102_2