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MATERIAL COMPLEMENTARIO
ENHANCING PERCEPTUAL REASONING THROUGH ACTIVE CALCULUS INSTRUCTION: A NEUROPSYCHOLOGICAL PILOT STUDY IN ENGINEERING EDUCATION

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Palavras-chave: *Cálculo; Educação; Funções cognitivas; Habilidades visuoespaciais.*

A B S T R A C T

Calculus, a cornerstone of mathematics that explores continuous change through differentiation and integration, is pivotal for higher education in engineering and STEM careers. Yet, its instruction presents challenges, with many students grappling with its complexity, accentuating the need for innovative teaching strategies. Grounded in the nexus between calculus and higher cognitive functions, this research evaluates the outcomes of a student-centered approach in enhancing understanding and applying calculus and seeks to determine its impact on broader cognitive functions. Employing a pretest-retest design with control and intervention groups, we merge cognitive evaluations, including subtests from the Wechsler Adult Intelligence Scale, with calculus performance assessments. Our results revealed the intervention group exhibited enhanced calculus performance and improvement in the perceptual reasoning index, especially in block design subtest, reflecting enhanced visuospatial skills. This study offers valuable perspectives for improving calculus instruction and potentially improving students' cognitive capabilities by implementing a student-centered approach.

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R E S U M E N

El Cálculo, piedra angular de las matemáticas que estudia el cambio continuo mediante la diferenciación y la integración, es fundamental en la educación superior en ingeniería y carreras STEM. Sin embargo, su enseñanza presenta desafíos, ya que muchos estudiantes lidian con su complejidad, lo que acentúa la necesidad de estrategias innovadoras de instrucción. Basada en la relación entre el cálculo y las funciones cognitivas superiores, esta investigación evalúa los resultados de un enfoque centrado en el estudiante para mejorar la comprensión y aplicación del cálculo, y busca determinar su impacto en funciones cognitivas más amplias. Utilizando un diseño de pretest-retest con grupos control e intervención, combinamos evaluaciones cognitivas, incluyendo subpruebas de la Escala de Inteligencia de Wechsler para Adultos, con pruebas de desempeño en cálculo. Nuestros resultados revelaron que el grupo de intervención mostró un mejor desempeño en cálculo y mejoras en el índice de razonamiento perceptivo, especialmente en la subprueba de diseño con cubos, lo que refleja un fortalecimiento de las habilidades visoespaciales. Este estudio ofrece perspectivas valiosas para optimizar la enseñanza del cálculo y, potencialmente, para favorecer el desarrollo de las capacidades cognitivas de los estudiantes mediante la implementación de un enfoque centrado en el estudiante.

R E S U M O

O Cálculo, pedra angular da matemática que estuda a mudança contínua por meio da diferenciação e da integração, é fundamental no ensino superior em engenharia e carreiras STEM. No entanto, seu ensino apresenta desafios, pois muitos estudantes enfrentam dificuldades com sua complexidade, o que acentua a necessidade de estratégias inovadoras de instrução. Com base na relação entre o cálculo e as funções cognitivas superiores, esta pesquisa avalia os resultados de uma abordagem centrada no estudante para aprimorar a compreensão e a aplicação do cálculo, e busca determinar seu impacto em funções cognitivas mais amplas. Utilizando um delineamento de pré-teste e pós-teste com grupos controle e intervenção, combinamos avaliações cognitivas, incluindo subtestes da Escala de Inteligência Wechsler para Adultos, com provas de desempenho em cálculo. Nossos resultados revelaram que o grupo de intervenção apresentou melhor desempenho em cálculo e melhorias no índice de raciocínio perceptivo, especialmente no subteste de construção com blocos, refletindo um fortalecimento das habilidades visuoespaciais. Este estudo oferece perspectivas valiosas para otimizar o ensino do cálculo e, potencialmente, favorecer o desenvolvimento das capacidades cognitivas dos estudantes por meio da implementação de uma abordagem centrada no aluno.

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SUPPLEMENTARY MATERIAL

Examples of the calculus tests

Below are examples of items used in the calculus tests to assess performance in calculus skills throughout the course according to the learning outcomes.

LEARNING OUTCOME: *i)* Apply the properties of real numbers:

1) Find the solution set for the inequality $2 \leq |x^2 - 4x + 2|$

Cognitive Functions Required:

- **Working Memory**: This task requires students to retain and manipulate multiple pieces of information about the properties of real numbers and inequalities to find the solution set.
- **Processing Speed:** Quickly evaluating the steps involved in solving the inequality.
- **Perceptual Reasoning:** Visualizing the solution set on a number line and understanding the absolute value function.
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LEARNING OUTCOME: *ii)* Solve problems involving conic curves:

1) A tunnel through a mountain is constructed such that its opening is elliptical. The road that will pass through the tunnel will have two lanes: each 4 meters wide. If the maximum height of the tunnel is to be 5 meters, and the height on each side of the road must be 4 meters, what should be the width of the tunnel's base? What is the height of a vertical pillar located 6 meters from the center of the road?

Cognitive Functions Required:

- **Perceptual Reasoning**: Students need to visualize the elliptical shape of the tunnel and apply geometric principles to solve for dimensions, using their spatial reasoning skills.
- **Working Memory**: Holding the dimensions and properties of the ellipse while performing calculations.
- **Processing Speed**: Efficiently performing geometric calculations and adjustments.

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LEARNING OUTCOME: *iii)* Calculate the limit of indeterminate forms of real functions in one variable:
An industry dedicated to salmon farming introduced 50 specimens into a breeding pool. It is established that the number of salmons will grow according to the model:

$$N(t) = \frac{10(5+3t)}{1+0.04t}$$

Where *t* represents time in years.

1) Calculate the projection for the number of specimens after 2 and 4 years.
2) As the years go by, what is the projection for the number of specimens in the pool?

Cognitive Functions Required:

- **Fluid Intelligence**: This requires logical reasoning and problem-solving to understand and apply the growth model for salmon population over time.
- **Working Memory:** Retaining the growth model and intermediate calculations.
- **Processing Speed:** Quickly applying the model to compute projections.
- **Perceptual Reasoning:** Understanding the growth trend and visualizing the limit as time approaches infinity.

LEARNING OUTCOME: *iv)* Determine injectivity and/or continuity of real functions in one variable.
Analyze the continuity of the function f at the points $x_0 = 0$ and $x_0 = \frac{1}{2}$, where:

$$f(x) = \{\frac{tan(x)}{2x}\ x < 0\ 1 - x\ 0 \le x \le \frac{1}{2}\ \frac{\sqrt{4x^2+3}-2}{2x-1}\ x > \frac{1}{2}$$

Cognitive Functions Required:

- **Attention/Inhibitory Control:** Students must focus on relevant properties of continuity at specific points and ignore extraneous details, demonstrating the ability to inhibit irrelevant information.
- **Working Memory**: Retaining the function's definitions and conditions.
- **Processing Speed:** Quickly evaluating limits and checking continuity.
- **Perceptual Reasoning**: Visualizing the function's behavior at the specified points.

LEARNING OUTCOME: *v)* Calculate the derivative of real functions in one variable:

1) Calculate the derivative of the function $f(x) = log\ log\ \left(\frac{sin(3x)}{x^2}\right)$
2) Determine the equation of the tangent line to the function $y = y(x)$ defined implicitly by the curve $x^3y^2 + ye^x + sin(x+y) = 0$ at the point (0,0).

Cognitive Functions Required:

- **Fluid Intelligence**: Students need to apply differentiation rules and implicit differentiation techniques, engaging their problem-solving and logical reasoning skills.
- **Working Memory**: Holding the rules of differentiation and intermediate results.
- **Processing Speed**: Efficiently performing differentiation steps and implicit differentiation.
- **Perceptual Reasoning**: Visualizing the curve and the tangent line at the given point.

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LEARNING OUTCOME: *vi)* Interpret the derivative in physical and geometric problems:
From a height of 16 meters above ground level, an object is thrown upwards, following the function $s(t) = -t^2 + 6t + 16$, where *t* is the time in seconds.

1) What is the maximum height reached by the object?
2) What are the velocity and acceleration of the object 2 seconds after being thrown?
3) How much time will it take for the object to reach the ground?

Cognitive Functions Required:

- **Perceptual Reasoning and Working Memory:** This involves visualizing the parabolic motion of the object and its trajectory described by the function and performing multiple calculations to determine height, velocity, and acceleration, enhancing both spatial reasoning and information retention and manipulation.
- **Processing Speed**: Quickly differentiating the function to find velocity and acceleration.

LEARNING OUTCOME: *vii)* Sketch the graph of real functions in one variable:

Given the function $f(x) = \frac{x^3-1}{x}$, determine:

1) The domain of the function
2) Intersections with the coordinate axes
3) Horizontal and/or vertical asymptotes
4) Intervals of monotonicity
5) Maxima and/or minima
6) Intervals of concavity
7) Inflection points
8) Sketch the graph

Cognitive Functions Required:

- **Cognitive Flexibility**: Students must switch between different representations and analyses of the function (algebraic, graphical, numerical) to fully understand and sketch the graph.
- **Working Memory**: Retaining various properties and calculations for the function.
- **Processing Speed**: Efficiently analyzing and plotting the function's characteristics.
- **Perceptual Reasoning:** Visualizing the graph and its features based on the analysis.

LEARNING OUTCOME: *viii)* Solve optimization problems of real functions in one variable:
A sheet of paper must contain 18 cm^2 of text. The lateral margins of the sheet should be 1 cm, and the top and bottom margins should be 2 cm.

1) Calculate the length and width of the sheet so that the minimum amount of paper is used.

Cognitive Functions Required:

- **Planning and Problem-Solving**: Students need to plan and execute a multi-step solution to optimize the dimensions of the sheet, engaging their strategic thinking and organizational skills.
- **Working Memory**: Holding the constraints and equations involved in the optimization problem. **Processing Speed**: Quickly differentiating and solving the optimization problem.
- **Perceptual Reasoning**: Visualizing the dimensions and layout of the sheet to minimize paper usage.

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Supplementary Tables

Table S1. Examples of how cognitive functions linked to each major topic in calculus:

1. Elements of Real Numbers

Working Memory	Solving exercises that require holding multiple properties of real numbers in mind while solving problems. For instance, students can be asked to classify and order sets of real numbers (e.g., rationals, irrationals) based on their properties, enhancing their ability to retain and manipulate information.
Processing Speed	Quickly evaluating the steps involved in solving inequalities.
Perceptual Reasoning	Visualizing the solution set on a number line and understanding the absolute value function.

Example: Provide a list of mixed real numbers and ask students to categorize them as rational or irrational, and then order them on a number line.

2. Conics

Perceptual Reasoning	Using visual representations of geometry forms, students visualize the shapes and properties of conic sections. This engages their ability to understand and work with geometric representations.
Working Memory	Holding the dimensions and properties of visual representations while performing calculations.
Processing Speed	Efficiently performing geometric calculations and adjustments.

Example: Students use software like GeoGebra to explore the intersection of a plane with a double-napped cone to form parabolas, ellipses, and hyperbolas, allowing them to see the effects of changing parameters in real-time.

1. Functions

Cognitive Flexibility	Students switch between different representations of functions (e.g., graphical, algebraic, numerical). This helps them understand the connections between various forms and engages their problem-solving flexibility.
Working Memory	Retaining various forms of the function and intermediate calculations manipulating information to solve problems.
Processing Speed	Quickly analyzing and converting between different representations of functions, and efficiently computing values and solving related problems.
Perceptual Reasoning	Understanding the behavior and characteristics of functions by visualizing their graphs and interpreting various features such as trends, slopes, intercepts, and asymptotes. This involves recognizing patterns and relationships within the function's graphical representation.

Example: Presenting a function, students represent it graphically, then derive the algebraic expression from the graph, and finally create a table of values. In the test presented in the supplementary material, this requires logical reasoning and problem-solving to understand and apply the growth model for the salmon population over time.

2. Limits and Continuity

Inhibitory Control	Focusing on relevant details and ignoring irrelevant information when analyzing limits and continuity. This is implemented through problems that include extraneous data or distractions.
Working Memory	Retaining different expressions for the function in various intervals while analyzing limits and continuity at specific points.
Processing Speed	Quickly evaluating the limits from the left and right at specific points and

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	determining the function values at these points.
Perceptual Reasoning	Visualizing the function's behavior near the points of interest and understanding the changes in the function's form across different intervals.

Example: In a complex scenario with unnecessary information, students determine the limit of a function at a point, focusing only on the relevant aspects of the problem.

3. Derivatives

Fluid Intelligence	Engaging in real-world problems that require the application of derivatives to solve, students apply their abilities to transfer learning to novel situations. This promotes logical reasoning and problem-solving skills.
Working Memory	Holding the rules of differentiation and intermediate results.
Processing Speed	Efficiently performing differentiation steps and implicit differentiation.
Perceptual Reasoning	Visualizing the curve and the tangent line at the given point.

Example: In a physics problem, students need to use derivatives to calculate the velocity and acceleration of an object at different times, interpreting the physical meaning of these derivatives.

4. Applications of Derivatives

Planning	Developing multi-step plans to solve optimization problems or related rates problems. This improves their ability to organize and execute a sequence of steps methodically.
Working Memory	Retaining various constraints and equations involved in the optimization problem.
Processing Speed	Quickly differentiating and solving the optimization problem.
Perceptual Reasoning	Visualizing the dimensions and layout of the objects being optimized.

Example: Solving an optimization problem, such as finding the dimensions of a box with maximum volume given a fixed surface area, students need to outline and follow a step-by-step plan to derive and solve the necessary equations.

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Table S2. Two-Way ANOVA for the Cognitive Assessment Pretest and subsample groups

Source of Variation	SS (Type III)	df	MS	F (DFn, DFd)	p-value
Interaction	72.21	9	8.023	F (9, 490) = 0.1389	P=0.9986
Subtest Factor	146378	9	16264	F (9, 490) = 281.6	P<0.0001
Group Factor	6.791	1	6.791	F (1, 490) = 0.1176	P=0.7318
Residual	28297	490	57.75		

SS = Sum of Square; MS = Mean Square

Table S3. Two-Way ANOVA for the Cognitive Assessment Control Group Pretest-Retest

Source of Variation	SS (Type III)	df	MS	F (DFn, DFd)	p-value
Interaction	443.5	9	49.28	F (9, 420) = 0.8146	P=0.6029
Subtest Factor	143270	9	15919	F (9, 420) = 263.2	P<0.0001
Group Factor	530.5	1	530.5	F (1, 420) = 8.769	P=0.0032
Residual	25407	420	60.49		

SS = Sum of Square; MS = Mean Square

Table S4. Two-Way ANOVA for the Cognitive Assessment Intervention Group Pretest-Retest

Source of Variation	SS (Type III)	df	MS	F (DFn, DFd)	p-value
Interaction	767.2	9	85.25	F (9, 560) = 1.580	P=0.1178
Subtest Factor	186376	9	20708	F (9, 560) = 383.8	P<0.0001
Group Factor	1115	1	1115	F (1, 560) = 20.66	P<0.0001
Residual	30216	560	53.96		

SS = Sum of Square; MS = Mean Square

Table S5. Two-Way ANOVA for the Indexes Comparisons: Intervention Group Pretest-Retest

Source of Variation	SS (Type III)	df	MS	F (DFn, DFd)	p-value
Interaction	1064	2	532.2	F (2, 168) = 1.760	P=0.1752
Subtest Factor	125556	2	62778	F (2, 168) = 207.6	P<0.0001
Group Factor	3738	1	3738	F (1, 168) = 12.36	P=0.0006
Residual	50799	168	302.4		

SS = Sum of Square; MS = Mean Square

Table S6. Two-Way ANOVA for the Indexes Comparisons: Control Group Pretest-Retest

Source of Variation	SS (Type III)	df	MS	F (DFn, DFd)	p-value
Interaction	425.3	2	212.7	F (2, 126) = 0.6508	P=0.5234
Subtest Factor	99646	2	49823	F (2, 126) = 152.5	P<0.0001
Group Factor	1817	1	1817	F (1, 126) = 5.560	P=0.0199
Residual	41174	126	326.8		

SS = Sum of Square; MS = Mean Square

Table S7. Two-Way ANOVA for the Cognitive Assessment Pretest and subsample groups

Source of Variation	SS (Type III)	df	MS	F (DFn, DFd)	p-value
Interaction	232.4	9	25.82	F (9, 490) = 0.4631	P=0.8992
Subtest Factor	178214	9	19802	F (9, 490) = 355.1	P<0.0001
Group Factor	14.80	1	14.80	F (1, 490) = 0.2655	P=0.6066
Residual	27325	490	55.77		

SS = Sum of Square; MS = Mean Square